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Criteria for choosing observables in elementary processes

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Abstract. The increasing versatility of polarisation experiments in particle and nuclear physics has brought into focus theoretical problems associated with the optimum extraction of information from experimental results. One such problem is the selection of observables whose measurement would lead to the unambiguous determination of reaction amplitudes of interest. This paper presents a solution of the problem in the form of general criteria using certain easily constructed diagrams.

1. Introduction

Over the past several years, polarisation experiments in nuclear and particle physics have become increasingly versatile and are beginning to yield significant information, particularly about the non-dynamical structure of reaction amplitudes. In the absence of a reliable dynamical theory, it is useful to extract as much model-independent information as possible from the experimental results. Now, for quantum systems, measurable quantities are invariably expressed as linear combinations of bi-linear products of certain amplitudes while theories deal with the amplitudes themselves. One is thus naturally led to enquire about the general criteria for selecting observables whose measurement would make it possible to deduce the amplitudes of interest almost unambiguously, i.e. without any continuum of ambiguity.

A diagrammatic approach to such problems was first introduced by Moravcsik and Yu (1969) and discussed further by Goldstein *et al* (1974). The prescription developed in those papers utilised the characterisation of complex amplitudes and their bi-linear products by real and imaginary parts. The advantage of using instead a phase-magnitude decomposition of complex quantities in this context was first realised and put forward by the present author (Jameel 1976). The purpose of this paper is to present a somewhat different formalism which is also general and applicable to reactions among elementary systems having abritrary spin.

2. Statement of problem

Let us consider a reaction which is completely described by n complex amplitudes $a_1, a_2, \ldots, a_n$. Physically observable quantities may involve a number of bi-linear products of the type $a_ia_j^*$. Instead of dealing with the real and imaginary parts of a_i and

$a_i a_j^*$ in the manner of Goldstein *et al* (1974), we decompose these complex quantities into their magnitudes and phases as follows:

$$a_i \equiv A_i \exp(\mathrm{i}\phi_i) \tag{1}$$

$$a_i a_j^* \equiv A_{ij} \exp(\mathrm{i}\phi_{ij}). \tag{2}$$

These relations serve to define the real quantities A_i, A_{ij}, ϕ_i and ϕ_{ij}. It is immediately obvious that

$$A_{ij} = A_{ji} = A_i A_j \tag{3}$$

$$\phi_{ij} = -\phi_{ji} = \phi_i - \phi_j. \tag{4}$$

Adopting nomenclature similar to existing literature, the magnitudes A_{ij} and phases ϕ_{ij} are called *bicoms* as they involve information about two amplitudes. Further, a bicom set is said to *belong* to a set of amplitudes if the bicoms together carry all the indices, and no others, which appear on the amplitudes. For example, the bicom set $\{A_{12}, A_{23}, \phi_{13}\}$ belongs to the amplitude set $\{a_1, a_2, a_3\}$ while the bicom set $\{A_{23}, \phi_{34}\}$ does not so belong because the index 4 does not appear on any of the amplitudes. Again, a set of bicoms belonging to a set of amplitudes is said to be *good* if the former set determines the latter set without any continuum of ambiguity.

Recalling that the actual problem is the determination of amplitudes from observables, a word may be added concerning the introduction of an intermediate quantity 'bicom' in the discussion. The point is that, while an experimental observable may involve any number of amplitudes, a bicom by definition depends on no more than two different amplitudes. Thus it turns out to be simpler to analyse the determination of amplitudes from bicoms rather than from observables. Furthermore, nothing essential is lost by not dealing directly with observables, because the relationship between observables and bicoms is known and usually quite simple so that there is little difficulty in going from bicoms to observables or conversely.

Now among the set of amplitudes $\{a_1, a_2, \ldots, a_n\}$ an overall phase can always be chosen arbitrarily. In order, therefore, to determine the n amplitudes, measurement of $(2n-1)$ independent observables must be carried out for each value of the kinematic variables. These $(2n-1)$ observations in turn imply the knowledge of $(2n-1)$ real bicoms. If we know fewer than $(2n-1)$ values, there will be a continuum of sets of amplitudes compatible with the bicoms. As we increase the number of bicoms, the extent of the ambiguity in amplitudes decreases until, with $(2n-1)$ functionally independent bicoms, there ceases to be a continuum of valid solutions for the amplitudes. Any remaining discrete ambiguities may be removed by obtaining some additional information.

The problem studied in this paper may now be stated as the following question. What are the necessary and sufficient conditions that a set of bicoms belonging to a set of amplitudes determines those amplitudes without any continuum of ambiguity? For clarity and convenience in application, the answer to this question will be given in the form of a theorem detailing the conditions from an operational point of view.

3. The goodness criteria

One starts with the amplitudes needed to describe a certain elementary process completely and the set of bicoms (obtained from experiments) whose 'goodness' is being investigated. A diagram is then drawn using the following rules:

(i) denote each amplitude by a point numbered by the index of that amplitude;

(ii) for each known $A_{ij}(\phi_{ij})$ draw a full (broken) line joining point i to point j;

(iii) for all known magnitudes of the type A_{ii}, put a cross $\times$ on the corresponding points; also put crosses on vertices of odd-sided polygons formed by full lines.

The criteria for goodness may then be stated as follows.

Theorem. A bicom set belonging to n amplitudes is good if and only if, in the diagram drawn according to the above prescription, the following conditions are fulfilled:

C(1) there exists a broken-line path between any pair of points;

C(2) points which are not crossed are connected to at least one crossed point through a full-line route.

Proof. The n amplitudes are unambiguously determined if their phases and magnitudes are known. The phase-magnitude decomposition allows one to study the two parameters separately.

(*a*) *Phases of the amplitudes.* As discussed earlier, the phase of any one amplitude, say a_k, can be chosen arbitrarily. Since, by condition C(1), every point is connected to the point k through broken lines, the difference in phase between a_k and any other amplitude can be deduced from the known ϕ. Thus the phases of all amplitudes are determined with respect to the phase of a_k. This proves that C(1) is a sufficient condition for 'goodness' of the diagram. Necessity is also evident because if even one pair of points is not connected by a broken line route, their relative phase will remain indeterminate.

(*b*) *Magnitudes of the amplitudes.* Let us show that if a point is crossed, the corresponding amplitude is determined in absolute value. This is obvious if the cross has been placed on account of $A_{ii} \equiv A_i^2$ being known from experiment. It is also easy to see that, if a closed full-line polygon is formed among any k points, it signifies the knowledge of, say, A_1A_2, $A_2A_3, \ldots, A_{k-1}A_k$, A_kA_1. From the last $(k-1)$ products, one can eliminate $A_3, \ldots, A_k$ obtaining finally the ratio A_1/A_2 *provided k is odd.* The magnitude A_1 is thereby determined, leading to the knowledge of the other magnitudes A_2 through A_k. Note that, if k is even, the elimination will ultimately lead back to the product A_1A_2 which is redundant information. This proves that points which are crossed correspond to amplitudes whose magnitudes are known.

Now, let us consider points which are not crossed. If condition C(2) is fulfilled, every uncrossed point is linked to at least one crossed point via full lines. The situation will then be similar to figure 1 where the magnitude A_i is known and the uncrossed point m is under discussion. Since A_i and the products A_iA_j, A_jA_k, A_kA_l and A_lA_m are known, it is easily seen that all the amplitudes A_j, A_k, A_l and A_m can be determined.

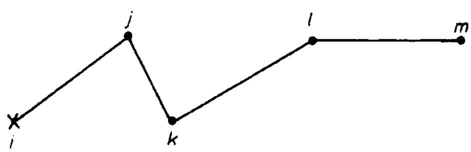

Figure 1. An illustration of condition C(2).

Hence C(2) is a sufficient condition for determination of magnitudes of all the amplitudes. It is also necessary because if all points are linked by full lines but no point is crossed, then the magnitudes can be determined only up to an arbitrary factor. Again, if a given uncrossed point is isolated from any crossed point, there is no other way in which the corresponding magnitude can be determined. As illustration, figure 2(a) shows a diagram corresponding to a 'good' set of bicoms while figure 2(b) represents a set of bicoms from which amplitudes a_5 and a_6 cannot be determined unambiguously.

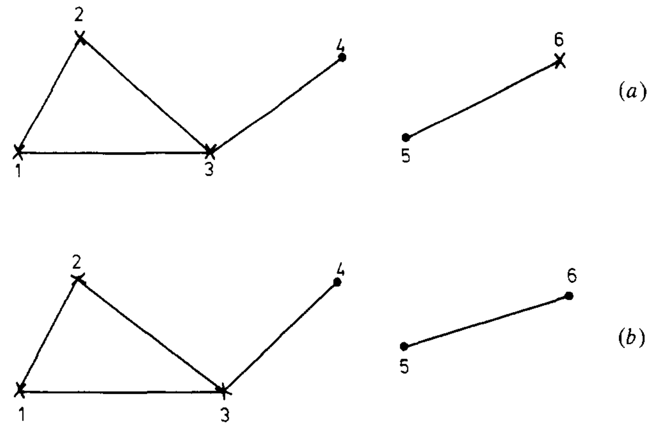

Figure 2. (a) The magnitude of all amplitudes can be determined. (b) There is a continuum of ambiguity for amplitudes corresponding to points 5 and 6.

4. Concluding remarks

In conclusion, it may be remarked that the present analysis differs from the work of other authors (Moravcsik 1969, Goldstein *et al* 1974) essentially in characterising the amplitudes and their bi-linear products by their magnitudes and phases rather than real and imaginary parts. This may make contact with observables somewhat less direct, although in certain recent formalisms (Sukhatme *et al* 1975), phase-magnitude decomposition is actually more convenient to use. In any case, the diagrammatic formalism presented here has the advantage that the 'goodness' of a set of bicoms can be determined by mere inspection of a diagram or, at most, by following a few simple operational steps. By contrast, the schemes evolved earlier entail the examination of all subsets of k points ($k < n$) as well as the consideration of all possible i-transforms (Goldstein *et al* 1974) of the diagram.

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